

Long-term memory in the Irish market (ISEQ): evidence from wavelet analysis.

Adel Sharkasi, Heather J. Ruskin and Martin Crane

School of Computing, Dublin City University

Email: asharkasi, hruskin and mcrane@computing.dcu.ie

(Dated: February 2, 2008)

Abstract

Researchers have used many different methods to detect the possibility of long-term dependence (long memory) in stock market returns, but evidence is in general mixed. In this paper, three different tests, (namely Rescaled Range (R/S), its modified form, and the semi-parametric method (GPH)), in addition to a new approach using the discrete wavelet transform, (DWT), have been applied to the daily returns of five Irish Stock Exchange (ISEQ) indices. These methods have also been applied to the volatility measures (namely absolute and squared returns). The aim is to investigate the existence of long-term memory properties. The indices are Overall, Financial, General, Small Cap and ITEQ and the results of these approaches show that there is no evidence of long-range dependence in the returns themselves, while there is strong evidence for such dependence in the squared and absolute returns. Moreover, the discrete wavelet transform (DWT) provides additional insight on the series breakdown. In particular, in comparison to other methods, the benefit of the wavelet transform is that it provides a way to study the sensitivity of the series to increases in amplitude of fluctuations as well as changes in frequency. Finally, based on results for these methods, in particular, those for DWT of raw (or original), squared and absolute returns, it can be concluded that there is strong indication for persistence in the volatilities of the emerging stock market returns for the Irish data.

keywords: long-term memory, classical and modified R/S methods, GPH test and the Discrete Wavelets Transform.

PACS numbers: 89.65.Gh, 05.10.Gg, 05.10.-a.

I. INTRODUCTION

There is no unique definition of long-term memory (**LTM**) processes (first introduced in [1]) which measure long range dependence. Such a process is generally defined as a series having a slowly declining correlogram or an infinite spectrum at zero frequency (see [2]).

The existence of long-range dependence (**LRD**) in a stock market has been an important topic in recent financial research as LTM models are able to describe features of data of various granularities using the same parameters. Several studies find evidence of long memory in stock market returns. ([3], [4] and [5]), while others show that there is none or at best there is weak evidence ([6], [7], [8], [9], [10] and [11]).

In [12], the author utilized GPH, classical and modified Rescaled Range (R/S) methods to test the presence of LTM in the international stock index returns of the G-7 countries and found no evidence of LTM in these series with the exception of West Germany. Further, the authors, in [13], applied the Spectral Regression Method to test for long-range dependence in 10 U.S stock returns, and returns of 30 firms included in the Dow Jones Industrial Index. Their results showed no evidence of long memory in these returns, as a whole, but some evidence for persistence in five companies while three other firms exhibited anti-persistence. In [14], the authors used a Lagrange Multiplier procedure and reported that long memory exists in the squared returns but not in the returns themselves.

More recently, Elekdag [15] applied GPH methods to volatilities of a large data set of emerging markets and found strong evidence for LTM in these series. This evidence was robust to various volatilities, specifically the absolute and modified log-squared returns. Further, Sibbertsen [16] found significant evidence of LTM in the volatilities of several German stock returns, investigated using the classical and tapered log-periodogram regression methods. Several articles detail the fact that emerging capital markets are more likely to have LTM than the major capital markets ([17], [18], [19], [20] and [21]).

It seems clear from the literature, therefore, that the volatilities of stock returns, such as squared and absolute returns, are more likely to have LTM than the stock returns themselves. The aim of this paper is twofold; (1) To employ the discrete wavelet transform (**DWT**) (for description see, [22], [23] and [24]) as a new testing approach to investigate the existence of LTM in the returns series and volatilities of five Irish indices. (2) To compare DWT with other methods [namely Rescaled Range (R/S) (introduced in [1]), its modified form

(for detail see [6]) and GPH (suggested in [25])). We examine Irish index returns, (namely Overall, Financial, General, Small Cap and ITEQ), in order to determine whether long memory behaviour is exhibited in any or all of these.

The remainder of this paper is organized as follows: In Section II the data and results are described and our conclusion is presented in section III.

II. DATA AND RESULTS

A. Data Overview

The data sets considered in this study are the daily closing values of five Irish Stock Exchange (ISEQ) indices, namely Overall, Financial, General (from 04/1/1988 to 30/9/2003), Small Cap (from 4/1/1999 to 30/9/2003) and ITEQ (from 4/1/2000 to 30/9/2003). The daily returns of all these indices are calculated as follows, [Daily Returns= $\ln(P_t/P_{t-1})$ where P_t and P_{t-1} are the index price at time t and $t - 1$ respectively].

B. Results

The R/S and Lo's R/S analysis are applied to the index returns and their volatility measures and the results are reported in Table I. There appears to be little evidence of long memory property in all returns series themselves (from either method). There is, however, strong evidence of long-range dependence in the absolute and squared returns of all indices except in those of the Small Cap index.

TABLE I: Results of the R/S analysis and Lo's modified R/S test.

ISEQ index	Series	V-test of R/S	V-test of Lo's R/S
Overall	Returns	1.7469	1.4776
	Absolute	7.2223**	4.3492**
	Squared	5.0001**	3.2941**
Financial	Returns	1.4493	1.3007
	Absolute	7.8112**	4.6176**
	Squared	5.9294**	3.7573**

TABLE I: Continued.

ISEQ index	Series	V-test of R/S	V-test of Lo's R/S
General	Returns	1.7150	1.4154
	Absolute	7.5615**	4.8269**
	Squared	4.2129**	3.0913**
Small Cap	Returns	1.3499	1.1666
	Absolute	1.3587	1.1042
	Squared	1.4102	1.0757
ITEQ	Returns	1.7038	1.6499
	Absolute	2.7325**	1.9878*
	Squared	2.1961**	1.6664

*Note: V-tests are calculated as $V_n = W_n/\sqrt{n}$. The acceptance or rejection of the null hypothesis at $\alpha\%$ level for 5% or 1% is determined by whether or not V_n is contained in the interval $[0.809, 1.862]$ or $[0.721, 2.098]$ respectively. Thus * and ** indicate statistical significance at the 5% and 1% respectively.*

The spectral regression procedure (GPH) is also applied to estimate d and to test the hypothesis ($H_0 : d = 0$ vs. $H_1 : d \neq 0$) for index returns and their volatilities. We report the GPH test for different values of $\alpha = 0.45, 0.50, 0.55, 0.60$ in order to measure the sensitivity of this test to the choice of m (where $m = n^\alpha$). A two-sided test is performed, by constructing a t -statistic with the theoretical variance of the spectral regression error equal to $\pi^2/6$, to test the statistical significant of the d estimates. The results are reported in Table II. Based on this analysis, there is no evidence of long-term memory in any of the returns series and there is strong indication of persistence in the absolute and the squared returns of all indices, except that of the Small Cap index. The GPH method shows that the squared returns of the General index have no long memory behaviour while both R/S tests show that long-range dependence is strongly exhibited in this series.

TABLE II: GPH estimation of fractional differencing parameter d for daily returns of Irish Stock Exchange (ISEQ) indices.

Index↓	Series	α			
		0.45	0.50	0.55	0.60
Overall	Returns	0.0523	0.0519	0.0428	0.1197
		(0.452)	(0.571)	(0.599)	(1.697)
	Absolute	0.5060	0.4380	0.4000	0.3650
		(4.371)**	(4.810)**	(5.600)**	(6.387)**
	Squared	0.3853	0.3333	0.2966	0.2663
		(3.256)**	(3.664)**	(4.150)**	(4.665)**
Financial	Returns	-0.0019	0.1096	0.1151	0.1289
		(-0.016)	(1.205)	(1.309)	(1.742)
	Absolute	0.5313	0.4720	0.3617	0.3384
		(4.586)**	(5.189)**	(5.060)**	(5.926)**
	Squared	0.3925	0.3754	0.3250	0.3065
		(3.388)**	(4.127)**	(4.548)**	(5.368)**
General	Returns	-0.0280	0.0508	0.0696	0.0972
		(-0.242)	(0.558)	(0.974)	(1.703)
	Absolute	0.3959	0.3615	0.3127	0.3098
		(3.417)**	(3.974)**	(4.375)**	(5.426)**
	Squared	0.2188	0.0969	0.0551	0.0877
		(1.889)	(1.066)	(0.771)	(1.537)
Small Cap	Returns	-0.0874	0.0787	0.0377	0.1369
		(-0.543)	(0.607)	(0.362)	(1.611)
	Absolute	-0.0189	0.1261	0.1647	0.1586
		(-0.118)	(0.972)	(1.579)	(1.867)
	Squared	0.1143	0.0209	0.0278	0.0153
		(0.710)	(0.161)	(0.267)	(0.180)

TABLE II: Continued.

Index↓	Series	α			
		0.45	0.50	0.55	0.60
ITEQ	Returns	0.1011	0.0609	-0.0192	0.0729
		(0.576)	(0.436)	(-0.170)	(0.786)
	Absolute	0.5370	0.4723	0.4081	0.3169
		(3.061)**	(3.371)**	(3.621)**	(3.413)**
	Squared	0.4161	0.3411	0.2989	0.2316
		(2.372)*	(2.435)*	(2.651)**	(2.495)*

*Note: The d estimates (**bold**) corresponding to GPH of α . The t -tests are given in parentheses and their statistical significance are indicated by * and ** at the 5% and 1% significance level respectively.*

While these more conventional analyses are useful, serving to contrast the Irish with other markets' data, we now consider the relatively novel approach using the discrete wavelet transform (DWT) to analyze the volatility more directly. The DWT with symmlet 8 wavelet (s_8) for 6 levels (scales) is computed for daily returns series and their volatility measures (namely squared and absolute returns) of all Irish indices in order to investigate the long-term memory property. The DWT provides a more detailed breakdown of the contribution to the series energy from the high and low frequencies in the following manner. Table III (Panels: A, B and C) display the energy percentages for wavelet components (crystals) of the returns, squared and absolute, of Overall, Financial, General, Small Cap and ITEQ indices respectively. These percentages indicate the proportion of energy in these series explained by each wavelet crystal. From Table III (Panel A), it can be seen that high-frequency crystals (especially the first and the second) have much more energy than the lowest frequency one and this means that movements in the returns are mainly caused by the short-term fluctuations. This confirms that there is little evidence for long memory in the returns series. Table III (Panel B) shows that the lowest frequency component (s_6) of the squared returns of each of the Overall, Financial and ITEQ indices has more energy than the second high-frequency component (d_2) but less energy than the first crystal (d_1). This provides further detail on previous analysis and implies that movements in these squared returns are caused by *both* short-term and long-term fluctuations. Thus there is clear evidence of a long

TABLE III: Energy Percentages explained by each wavelet component.

Panel A: The daily returns of Irish indices.

Index \rightarrow	Overall	Financial	General	Small Cap	ITEQ
W.Crystals ↓					
d_1	0.433	0.431	0.447	0.493	0.476
d_2	0.239	0.251	0.236	0.234	0.210
d_3	0.158	0.163	0.138	0.093	0.181
d_4	0.079	0.074	0.083	0.078	0.055
d_5	0.036	0.033	0.037	0.045	0.036
d_6	0.029	0.024	0.029	0.023	0.027
s_6	0.026	0.024	0.030	0.035	0.014

Panel B: The squared returns of Irish indices.

Index \rightarrow	Overall	Financial	General	Small Cap	ITEQ
W.Crystals ↓					
d_1	0.367	0.326	0.388	0.314	0.317
d_2	0.162	0.182	0.188	0.234	0.185
d_3	0.121	0.115	0.116	0.217	0.125
d_4	0.074	0.059	0.118	0.047	0.069
d_5	0.046	0.047	0.044	0.042	0.049
d_6	0.026	0.021	0.024	0.031	0.016
s_6	0.205	0.251	0.122	0.116	0.240

memory property in the squared returns series. The lowest frequency component (s_6) of the squared returns of the General index has lower energy than the second highest frequency (d_2) but higher than that of the third component, indicative of a *weak* long memory effect in the squared returns of General index. However, the energy of the lowest frequency crystal of the squared returns of Small Cap index is even lower than that of the d_3 component and this clearly implies that the movements of this series are mostly caused by short-term fluctuations with no significant evidence of long-term memory.

Table III (Panel C), in contrast, illustrates a situation where the lowest frequency com-

Panel C: The absolute returns of Irish indices.

Index \rightarrow	Overall	Financial	General	Small Cap	ITEQ
W.Crystals\downarrow					
d_1	0.195	0.183	0.207	0.194	0.194
d_2	0.103	0.104	0.110	0.116	0.097
d_3	0.060	0.063	0.062	0.080	0.069
d_4	0.035	0.032	0.045	0.027	0.036
d_5	0.027	0.027	0.027	0.022	0.024
d_6	0.015	0.013	0.015	0.015	0.008
s_6	0.565	0.579	0.533	0.546	0.571

ponent (s_6) has much more energy than both the first (d_1) and the second (d_2) components together, which is strong evidence of long-range dependence in the absolute returns series with movements in these series mostly caused by long-term fluctuations. From the wavelet analysis it is clear that the frequency patterns are demonstrably different for the respective series where large energy percentages, associated with high frequency components, implies short-term memory dominance and *vice versa*.

III. CONCLUSION

In this article, the discrete wavelet transform (DWT) and three other methods, were employed to test for the presence of long memory in the five Irish Stock Exchange (ISEQ) indices. In agreement with findings for other indices, (e.g. [26], [15] and [16]), there is no evidence of long memory for returns series, while for squared and absolute returns, such a property does appear to exist. The exception is the Small Cap index for the Irish data, which shows no significant evidence of long-term dependence for any returns series. The DWT analysis, however, provides additional insight on the series breakdown. In particular, in comparison to other methods, the benefit of wavelet transform is that it provides a way to study the sensitivity of the series to increases in amplitude of fluctuations as well as changes in frequency. Finally, based on results for these methods, in particular, those for DWT of returns, squared and absolute returns, it can be concluded that there is strong indication for persistence in the volatilities of the emerging stock market returns for the Irish data.

-
- [1] H. E. Hurst, Transactions of the American Society of Civil Engineers **116**, 770 (1951).
- [2] C. W. Granger and Z. Ding, Journal of Econometrics **73**, 61 (1996).
- [3] C. F. Lee, G. M. Chen, and O. M. Rui, Journal of Financial Research **25**, 523 (2001).
- [4] S. Sadique and P. Silvapulle, International Journal of Finance and Economics **6**, 59 (2001).
- [5] A. Assaf and J. Cavalcante, European Review of Economics and Finance **4**, 3 (2005).
- [6] A. W. Lo, Econometrics **59**, 1279 (1991).
- [7] Y. W. Cheung and K. S. Lai, Journal of International Money and Finance **14**, 597 (1995).
- [8] B. Jacobsen, Journal of Empirical Finance **3**, 393 (1996).
- [9] C. Hiemstra and J. D. Jones, Journal of Empirical Finance. **4**, 373 (1997).
- [10] L. Berg and J. Lyhagen, Applied Financial Economics **8**, 435 (1998).
- [11] Z. Ding, C. W. Granger, and R. F. Engle, Journal of Empirical Finance **1**, 83 (1993).
- [12] N. Crato, Applied Financial Economics **4**, 33 (1994).
- [13] J. T. Barkoulas and C. F. Baum, Economics Letters **53**, 253 (1996).
- [14] I. N. Lobato and N. E. Savin, Journal of Business and Economic Statistics **16**, 261 (1998).
- [15] S. Elekdag, in *International Conference in Economics, The ISE Finance Award Series* (The Middle East Technical University, 2001).
- [16] P. Sibbertsen, in *Proceedings of International Conference on: Modelling Structural Breaks, Long Memory and Stock Market Volatility* (CASS Business School, London, UK, 2002).
- [17] G. Bekaert and C. R. Harvey, Journal of Finance **50**, 403 (1995).
- [18] J. H. Wright, International Finance Discussion Papers, The Federal Reserve Board (1999), available at <http://ideas.repec.org/p/fip/fedgif/650.html> [accessed on 19/06/2006].
- [19] G. C. Nath, in *Proceeding of Seventh Annual Capital Market Conference, Indian Institute of Capital Markets* (Mumbai, India, December 2001, 2001).
- [20] T. Henry, Applied Financial Economics **12**, 725 (2002).
- [21] J. Tolvi, Economic Bulletin **7**, 1 (2003).
- [22] H. S. Lee, in *Proceedings of ERF Eighth Annual Conference: Economic Research Forum*, (Cairo, Egypt, 2002).
- [23] A. Sharkasi, H. J. Ruskin, and M. Crane, International Journal of Theoretical and Applied Finance **8**, 603 (2005).

- [24] A. Sharkasi, M. Crane, H. J. Ruskin, and J. A. Matos, *Physica A: Statistical Mechanics and its Applications* **368**, 511 (2006).
- [25] J. Geweke and S. Porter-Hudak, *Journal of Time Series* **4**, 221 (1983).
- [26] J. Lee, T. S. Kim, and H. K. Lee, in *Proceedings of INFORMS/KORMS Seoul 2000 Conference* (Korea, June 2000, pages: 540-546, 2000).