Time and scale Hurst exponent analysis for financial markets

José A.O. Matos [a,b]

Sílvio M.A. Gama [c,b]
Heather J. Ruskin [d]
Adel Al Sharkasi [d,e]
Martin Crane [d]

[a] *Faculdade de Economia da Universidade do Porto, Portugal*

[b] *Centro de Matemática da Universidade do Porto, Portugal*

[c] *Faculdade de Ciências da Universidade do Porto Portugal*

[d] *School of Computing, Dublin City University, Ireland*

[e] *Department of Statistics, Faculty of Science, Garyounis University, Libya*

Abstract

We use a new method of studying the Hurst exponent with time and scale dependency. This new approach allows us to recover the major events affecting worldwide markets (such as the September 11th terrorist attack) and analyze the way those effects propagate through the different scales. The time-scale dependence of the referred measures demonstrates the relevance of entropy measures in distinguishing the several characteristics of market indices: "effects" include early awareness, patterns of evolution as well as comparative behaviour distinctions in emergent/established markets.

Key words: Long-term memory processes, Detrended fluctuation analysis, Hurst exponent, Econophysics
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1 Introduction

The main goal of this study is the analysis of stock exchange world indices searching for signs of coherence and/or synchronization across the set of studied markets.

We have expanded the scope of previous work on the PSI-20 (Portuguese Standard Index), since results there (3) seemed to provide a basis for a wider ranging study of coherence and entropy.

With that purpose we applied econophysics techniques related to measures of "disorder"/complexity (entropy) and a newly proposed (4) generalization of Detrended Fluctuation Analysis. As a measure of coherence among a selected set of markets we have studied the eigenvalues of the correlation matrices for two different set of markets **?**), exploring the dichotomy represented by emerging and mature markets and proposing a more refined classification. The indices are used to represent or characterise the respective market.

The classification of markets into mature or emergent is not a simple issue. The International Finance Corporation (IFC) uses income per capita and market capitalisation relative to Gross National Product (GNP) for classifying equity markets. If either (i) a market resides in a low or middle-income economy, or (ii) the ratio of the investable market capitalisation to GNP is low, then the IFC classifies the market as emerging, otherwise the classification is mature.

The data used in this study was taken daily for a set of worldwide market indices. As is usual in this kind of analysis (2) we base our results on the study of log returns $\eta_i = \log \frac{x_i}{x_{i-1}}$, where η_i is the log return at time step i.

2 Time and Scale Hurst exponent

2.1 Fractional Brownian motion

Fractional Brownian motion (fBm) is a well-known stochastic process where the second order moments of the increments scale as

$$E\{(X(t_2) - X(t_1))^2\} \propto |t_2 - t_1|^{2H} \tag{1}$$

with $H \in [0, 1]$. The Brownian motion is then the particular case where $H = 1/2$.

The exponent H is called the Hurst exponent. If $H < 1/2$, then the behaviour is anti-persistent (intermediate memory), that is, deviations of one sign are generally followed by deviations with the opposite sign. The limiting case $H = 0$, (short memory), corresponds to white noise, where fluctuations at all frequencies are equally present.

If $H > 1/2$, then the behaviour is persistent (long memory), i.e. deviations tend to keep the same sign. The limiting case $H = 1$, reflects $X(t) \propto t$, a smooth signal.

While motivation for using fBm was the fat-tail characteristic of real price distributions, this $H-$threshold for persistent/anti-persistent behaviour is useful in terms of determining when trends break down. For emergent markets, value of H is consistently higher than 0.5 **?**).

2.2 Detrended Fluctuation Analysis

The DFA (Detrended Fluctuation Analysis) technique consists in dividing a random variable sequence $X(t)$, of length s, into s/τ non-overlapping boxes, each containing τ points (5). The linear local trend $z(t) = a\,t + b$ in each box is defined to be the standard linear least-square fit of the data points in that box. The detrended fluctuation function F is then defined by:

$$F_k^2(\tau) = \frac{1}{\tau} \sum_{t=k\tau+1}^{(k+1)\tau} |X(t) - z(t)|^2 , \qquad k = 0, \cdots , \frac{s}{\tau} - 1. \tag{2}$$

Averaging $F(\tau)$ over the s/τ intervals gives the fluctuation $\langle F(\tau)\rangle$ as a function of τ. Here

$$\langle F^2(\tau)\rangle = \frac{\tau}{s} \sum_{k=0}^{s/\tau-1} F_k^2(t).$$

If the observable $X(t)$ are random uncorrelated variables or short-range correlated variables, the behaviour is expected to be a power law

$$\sqrt{\langle F^2(\tau)\rangle} \sim \tau^H . \tag{3}$$

2.3 Method characterisation

When we apply the DFA to a time series we compute a single real number (the estimated Hurst exponent) that describes the global behaviour. One of the possible generalisations is to evaluate the Hurst exponent for fixed size windows, thus measuring local Hurst exponents and from there studying their time dependency (see **? ?**)).

The general idea behind this method is the study of the local Hurst exponent as a function of both time and scale. In practical terms this method is a direct expansion of the "windowed" DFA applied in (3). Instead of fixing s we let it be a variable. The Hurst exponent, $H(t, s)$, for time t and scale s, is evaluated as the Hurst exponent obtained using the DFA , for the interval $[t - s/2, t + s/2]$.

Implications are wider than for a simple DFA. The general idea is to essentially invert the process and take $H(s, t)$ as the focus of the analysis with the DFA being an implementation detail. An alternative candidate technique for evaluating the Hurst exponent in the sub-intervals is, intuitively, the wavelet approach (6). In both cases H is recovered as a power of the scale, inside each sub-interval.

From the above condition we know that $s/2 + 1 \leq t \leq T - s/2$, where T is the time series length. In what follows, the maximum scale we consider is $s = T/4$, as for large scales, we essentially recover the Hurst exponent for the whole series.

A major concern in this work was to guarantee that exponents obtained through DFA were meaningful. For this reason we have used the same procedure as in (3), we have controlled the quality of the fits assuring that the regression coefficients of the linear least squares method were near unity for all studied markets, that was the reason why we have limited our analysis to scales (s) larger than 80 trading days observations. If we do not do this, the results would be unreliable, since the underlying time series is not well described by a fractional Brownian motion. To this combination of the DFA with time and scale dependency, we apply the term TSH (Time and Scale Hurst exponent).

2.4 Examples

Here we study some examples of the technique applied to several international markets. We choose these because they display details that are either unique feature or features common to several markets, in order to contribute to an understanding of the differences and similarities that TSH emphasises.

2.4.1 Nikkei

As an illustration of the method we worked with Nikkei 225 data ranging from 1990 to 2005. Nikkei was chosen because it is a well known and much studied financial index.

The graph resulting from application of the TSH method is shown in Figure 1. The graphic is represented as a contour plot, with exponents in range $[0.3, 0.9]$,

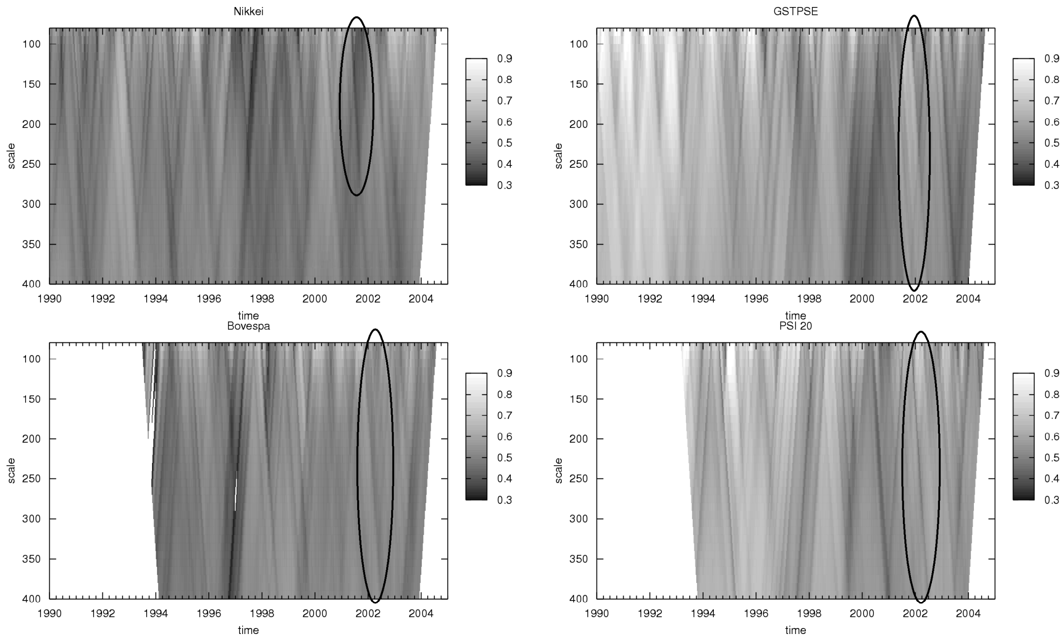

Figure 1. TSH applied to several markets. The scale (in trading days) is represented by the y-axis; the time is represented in x-axis (years).

for the series studied from 1990 − 2005 and the scale ranging between 100 and 400 trading days. In this work we adopted these fixed ranges since this representation permits ready comparison with other indices calculated.

In the Nikkei graphic (Figure 1) we can see that persistence is exhibited with the index normally around 0.5. This reflects a healthy borderline, of values near 0.5, and is to be expected since Tokyo is a mature market. In recent years we see a stripe that crosses all scales in year 2000, at the same time as the DotCom crash.

We have another stripe that starts in the fourth quarter of 2001 but does not go through all scales. Another period of high values of H starts for short scales in the third quarter of 2002, after a global crash and reaches large scales in 2004.

2.4.2 GSTPSE (Canada)

As seen in Figure 1, the market shows two distinct periods, before and after 1997. Before 1997 we see high values of the Hurst exponent over all scales. After that time, all the regions of high Hurst exponents are bounded in time and the background is as expect from a mature market, with the Hurst exponent around 0.5.

There are two stripes, for high values of H, after 1997 that cross all scales, one in 1998 and another starting around September 2001 and travelling forward for higher scales in time.

2.4.3 Bovespa (Brazil)

Bovespa, the São Paulo Stock Exchange Index, is known for its high volatility and is generally considered an emergent market. In Figure 1 we see erratic behaviour with H either near or above 0.5 and the corresponding stripes crossing together, back and forward in time, at all scales. There are two stripes, for high values of H, that start from short scales respectively in 1997 (Asian crashes) and 1998 (global crash) which merge for large scales. There is another, for $H > 0.5$, stripe that crosses all scales and starts for short scales around September 2001.

2.4.4 PSI-20 (Portugal)

Unmodified DFA, the predecessor of TSH, was applied to PSI-20 in (3). In Figure 1 we see the results of applying TSH to this market, from establishment of series in 1993.

Initial stages are both anti-persistent and subject to extreme values of the Hurst exponent. We can identify two stripes with a stable (higher) value of the Hurst exponent, during 1998, and another moving forward in time starting, for short scales, starting for September 2001. Notice that this stripe is so strong that it overlaps other stripes forming in the neighbourhood, implying an extreme event or major increase in entropy (high disorder and unpredictability).

The overall strength of the TSH is thus to provide a map of the evolution of markets to maturity, including significant episodes, but also gradual changes in response times, which are demonstrably different at initial and later stages.

2.5 Features

As can be seen in Figure 1 there are several notable features of the plots produced by TSH:

- We can distinguish mature markets by the stability [1] of H values around 0.5, most of the time and we can distinguish emergent markets by the stability of H values above 0.5 (again this agrees with other studies, see **?**)).

[1] Stability here refers to market at steady-state.

- For some periods, a phase transition appears to occur, sometimes observable across all scales, sometimes across partial scales only. This is reflected in the spikes which occur for both lower and large scales;
- *A priori*, we expected smooth variations of H for large scales since we are taking into account more data values and therefore we expect greater robustness to sudden changes of the data. This was already observed in the results obtained for PSI-20 and is confirmed by all the examples.
- Markets evolve in time, the Canadian case is a notable example of this, where we observe a shift from emergent to mature features. Although not so dramatic for all other cases we see over time a decrease in the values of the Hurst exponent.
- Significant events, causing marked change in the Hurst exponent behaviour, can be seen in almost all markets. September 11th 2001, the most striking recent example, can be seen in all Figures. Such features are easier to identify in more detailed, and coloured, graphics that can be found in **?**).
- Clearly studying the behaviour of the Hurst exponent for multiple time and scale intervals gives more refined detail on series component data The details obtained are richer than those obtained by calculating the Hurst exponent for the whole series and are indicative of the multifractal background from financial data (see (1)).

3 Results

3.1 Classification of global markets

It seems clear from the results, obtained from TSH, that we can distinguish different markets classes and can readily visualise the differences.. The most active, and mature, markets show a sustained behaviour, with $H \sim 0.5$, while the newer, emergent, markets show consistently higher values. There is evidence of further diversity, with hybrid markets moving between the two sustained state values.

The classification that we propose has thus three states:

(clearly) mature these market have H around 0.5. The presence of regions with higher values of H is limited to small periods and is well defined both in time and scale.

(clearly) emergent these market have H well above 0.5. The presence of regions with values of H around 0.5 is well defined both in time and scale.

hybrid unlike the two previous case the distinction between the mature and emergent phases is not well determined, with the behaviour seemingly mixing at all scales.

3.2 Data

We have considered, in this study, the major and most active markets worldwide from America (North and South), Asia, Africa, Europe and Oceania (see **?**)). All the data on the respective market indices are public and came from Yahoo Finance (`finance.yahoo.com`). We have considered the daily closure as the value for the day, to obviate any time zone difficulties.

The choice of the markets used in this study was determined by the aim to study major markets across the world in an effort to ensure that tests and conclusions could be as general as possible. Despite the breadth of the markets studied, results for a selection only are presented here for illustration.

4 Conclusions

We applied the TSH to study market evolution over time for a range of markets, variously classified as mature, emerging and hybrid, (with behaviour in the last category switching between the other two). TSH was used to compare the market set and to establish classes displaying similar behaviour at any given time. This classification allows us to distinguish major events that affect several markets, as opposed to local fluctuations in a single (or few) markets. Examples, reflected worldwide, include the Asian tiger crashes(1997), 9/11 in 2001 - cited above, the Madrid bomb attacks (2004), and others. The resulting classification is in agreement with that proposed by our wavelet analysis, proposed in (7).

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