CA659 Mathematical Models/Computational Science

In-Class Exercise 3

Exercises on Matrices

1. Find the determinants and eigenvalues of the following matrices:

   (a) $\begin{bmatrix} 2 & 2 \\ 5 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

   (b) $\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$

2. With the eigenvalues, we can find the eigenvectors of a matrix. An Eigenvector of a matrix A is any solution vector $\boldsymbol{x}$ for which: $A\boldsymbol{x} = \lambda\boldsymbol{x}$.

   Example: Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$

   (a) Firstly find the eigenvalues:
   Recall that the eigenvalues are calculated by solving $\det(A - \lambda I) = 0$
   (where *I* is the identity matrix). Thus
   $A - \lambda I = \begin{bmatrix} 1-\lambda & -2 \\ 1 & 4-\lambda \end{bmatrix}$
   $\det \begin{bmatrix} 1-\lambda & -2 \\ 1 & 4-\lambda \end{bmatrix} = 0$ gives the quadratic $(1-\lambda)(4-\lambda) + 2 = 0$
   which simplifies to $\lambda^2 - 5\lambda + 6 = 0$ hence $\lambda = 2, 3$ are the eigenvalues.

   (b) Now to find the eigenvectors: From above, these are the solution vectors to the system $A\boldsymbol{x} = \lambda\boldsymbol{x}$ when we substitute the eigenvalues above. Hence for $\lambda = 2$, we get:
   $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 2\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$
   So the first equation reads: $x_1 - 2x_2 = 2x_1$ giving $x_1 = -2x_2$ and thus the eigenvector $\boldsymbol{x}$ corresponding to the first eigenvalue λ is any multiple of $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$.
   In a similar way, we can find the second eigenvector as any multiple of $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

   Find the eigenvectors of the above matrices in Q1 using the method outlined above.