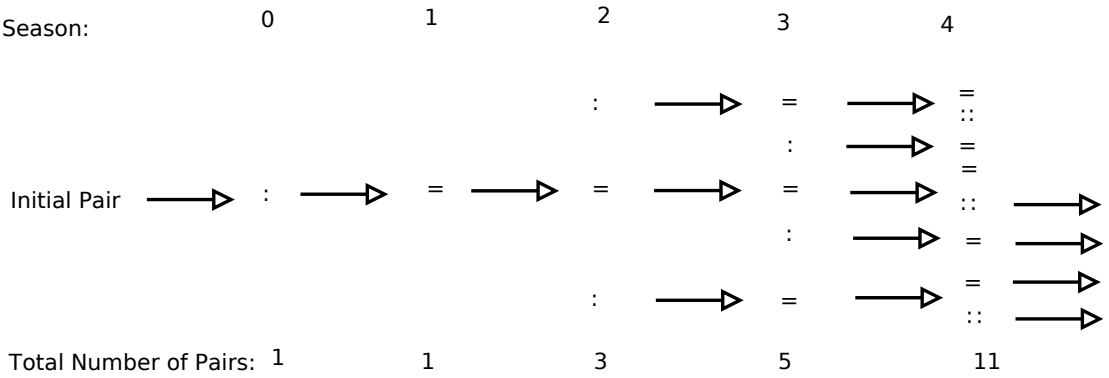
Solutions to Exercise 5

Q1 $M_{n+1} = M_n + 2M_{n-1}$ matrix equation is $\begin{bmatrix} M_{n+1} \\ M_n \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} M_n \\ M_{n-1} \end{bmatrix}$, $\lambda_{max} = 2.414$ so increases.

Diagram



Q2 For each wolf eating 12 rabbits, $\lambda_{max} = 1.02$ so population increases. If wolf eats 13 rabbits $\lambda_{max} = 1.01$ so population still increases. Critical predation rate is 14 rabbits per wolf.

Q3 For $L = \begin{bmatrix} 1 & 4.5 & 3.2 \\ 0.9 & 0 & 0 \\ 0 & 0.7 & 0 \end{bmatrix}$ the characteristic equation is given by $\lambda^2 - \lambda^3 + 4.05\lambda + 2.016 = 0$

by Newton-Raphson Method, can get $\lambda_{max} = 2.74$. Stable age distribution is given by $\begin{bmatrix} 70.8\% \\ 23.2\% \\ 5.9\% \end{bmatrix}$

Q4 Starting with $\begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.2 & 0.3 & 0 \\ 0.2 & 0.3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ it will take 9 years approx