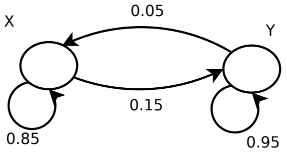
Solutions to Exercise 4

Q1 Transition Diagram is



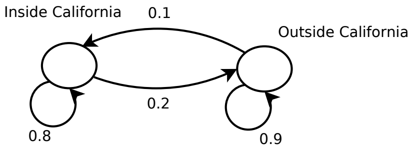
Can get the Transition Matrix by using

To \ From	X	Y
X	0.85	0.05
Y	0.15	0.95

Markov Transition Matrix is

M= $\begin{bmatrix} 0.85 & 0.05 \\ 0.15 & 0.95 \end{bmatrix}$. Stable state is given by $\begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix}$.

Q2 Transition Diagram is



Can get the Transition Matrix by using

To \ From	$Inside$	$Outside$
$Inside$	0.8	0.1
$Outside$	0.2	0.9

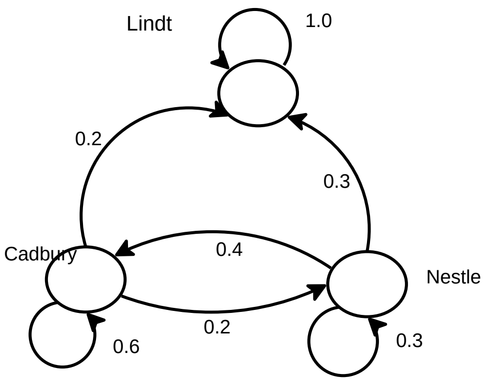
Markov Transition Matrix is M= $\begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}$. Stable state is given by $\begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$.

Q3 Can get the Transition Matrix by using

To \ From	N	C
N	0.6	0.2
C	0.4	0.8

Markov Transition Matrix is thus M= $\begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix}$. Can find from the associated characteristic equation that $\lambda_1 = 1$ and $\lambda_2 = 0.4$. Stable state is given by $\begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$.

Q4 The Transition Diagram is



When Lindt is added to the market the Transition Matrix M can be got from

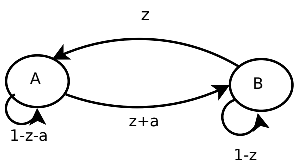
Fr	N	C	L
N	0.3	0.2	0
C	0.40	0.6	0
L	0.3	0.2	1

or $M = \begin{bmatrix} 0.3 & 0.2 & 0 \\ 0.4 & 0.6 & 0 \\ 0.3 & 0.2 & 1 \end{bmatrix}$. To find the steady state for this we can multiply M by $\frac{1}{3}\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ which is the stable state from Q3.

This gives $\frac{1}{3}\begin{bmatrix} 0.3 & 0.2 & 0 \\ 0.4 & 0.6 & 0 \\ 0.3 & 0.2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ which equates to $\begin{bmatrix} 0.23 \\ 0.53 \\ 0.23 \end{bmatrix}$ which is the state after 1 year

Do this two or three times and the vector will iterate towards $\begin{bmatrix} 0.14 \\ 0.32 \\ 0.54 \end{bmatrix}$. This is the new steady state.

Q5 Transition Diagram is given by



Can get the Transition Matrix by using

To \ $From$	A	B
A	$1-z-a$	z
B	$z+a$	$1-z$

Markov Transition Matrix is thus M= $\begin{bmatrix} 1-z-a & z \\ z+a & 1-z \end{bmatrix}$ QED

Stable state can be found from $\begin{bmatrix} 1-z-a & z \\ z+a & 1-z \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = 1.\begin{bmatrix} A \\ B \end{bmatrix}$ for some vector $\begin{bmatrix} A \\ B \end{bmatrix}$

Hence $(z+a)A + (1-z)B = B$

So $(z+a)A - zB = 0$ and so $A = \frac{z}{z+a}B$, making $\frac{1}{z+a}\begin{bmatrix} z \\ z+a \end{bmatrix}$ a steady state QED