CA659 Mathematical Models/Computational Science

In-Class Exercise 7

Problems On Differential Equations (cont'd)

1. The modelling of explosion/extinction of a population of crocodiles can be modelled according to the equation:

$$\frac{dP}{dt} = \frac{r}{M}P[P - M] \qquad (1)$$

Where:

$P(t)$, denotes number of crocodiles in the population after time *t* months;
M, denotes a threshold number of individuals in the population = 150 crocs;
r, the growth rate of the population is given by 0.06/month.

If $P = 200$ crocodiles initially, show (using Partial Fractions) that:

$$\frac{1}{P(P-M)} = \frac{1}{M}\left[\frac{1}{P-M} - \frac{1}{P}\right]$$

And hence show that the integral of (1) works out to be:

$$\frac{P-M}{P} = Ae^{rt} \qquad (2)$$

Find *A* and hence show that the population will crash after approx. 23 months

2. A culture of bacteria with population obeys the organic (Malthusian) model of growth, with reproductive rate *r*. A biologist harvests the bacteria from the culture at a constant rate *h*. Adjust the Malthusian model to account for the constant harvesting.

Perform a stability analysis (graph $\dot{\boldsymbol{P}}$ against $\boldsymbol{P}$, where a dot indicates differentiation with respect to time) and find the equilibrium point(s) in terms of *h* and *r.* Describe what will happen to the population in terms of the point(s) you have found. Describe giving reasons whether the equilibrium point(s) is/are stable or unstable.

Derive an expression for $P(t)$ in terms of *r*, *h* and P_0. Under what conditions will an initial population decay to zero? Find the time (in terms of *r*, *h* and P_0) required for this to happen.