CA659 Mathematical Models/Computational Science

In-Class Exercise 5

Revision Exercises on Difference Equations

1. In a genetically modified rabbit population, the rabbits now produce two pairs of offspring instead of the usual one. If the following assumptions are made:
   a. Each pair of rabbits can reproduce from two months old
   b. Each reproduction produces two pair of rabbits
   c. All rabbits survive.

   It is decided to model the above system using difference equations.
   Draw a diagram illustrating the reproduction of the rabbits for the first five time steps, differentiating between newborns and mature pairs, assuming there is one newborn pair at the initial time step. Hence derive an expression for the number of rabbits at time step *n+1* in terms of that at the previous time step *n*. Determine the eigenvalues of the system and say whether the population will increase or not.

2. The U.S. National Wildlife Foundation is trying to determine the importance of the rabbit population to that of the timber wolves, whose primary diet in Yellowstone National Park is the rabbit. Given the number of wolves at the beginning of year *k*, say w_k, and the number of hundreds of rabbits at the beginning of the year, say r_k, historical data indicate that the number of wolves at the beginning of the next year, w_{k+1}, and the number of hundreds of rabbits at the beginning of the next year, r_{k+1}, depend on w_k and r_k according to the following relationships:

   $$w_{k+1} = 0.72w_k + 0.2r_k$$
   $$r_{k+1} = -0.12w_k + 1.1r_k$$

   What will happen to the wolf and rabbit populations over a long period of time, assuming that nothing happens to change the relationships given by the above? What is the steady state solution of the system? Suppose that the situation outlined above changes and there is a predation rate of 13 rabbits per hundred. What does that do to the wolf and rabbit populations? What is the critical predation rate for the stability of the rabbit population?

3. You are given that a female population is divided into 3 equal age groups and that the average no. of female offspring for the first, second and third age groups are 1, 4.5 and 3.2 respectively. The proportion of the first and second age groups which survive to the next age group is 0.9 and 0.7 respectively. Set up a Leslie matrix for the population and hence calculate the long-term growth rate and population stable age distribution.

4. For the KulKola example in the notes, how long does it take for KulKola to take 90% of the Cola Market (Assuming that 5% revert to Pepsi and 5% to Coke)?