CA659 Mathematical Models/Computational Science

In-Class Exercise 4

Exercises on Markov Processes

1. Suppose there are two regional news shows in the local television viewing area, and we have conducted a survey of viewers to determine which the viewers have been watching. The first survey revealed that 40% of the viewers watched station X and 60% watched station Y. Subsequent surveys revealed that each week 15% of the X viewers switched to station Y and 5% of the Y viewers switched to station X. Draw a Transition Diagram and Transition matrix of this system and use Markov chains to make a prediction about the future television market from this information about the long-run distribution of viewers.

2. Each year 10% of the population of California move in and 20% of the people inside California move out. If y_k, z_k denote the number of people inside and outside the state in any year *k* respectively, starting with y_0 inside and z_0 outside the state, and $\boldsymbol{x}_k = \begin{bmatrix} y_k \\ z_k \end{bmatrix}$
    a. Write down a matrix model for the number of people inside and out at the end of a year in terms of those at the start.
    b. Show by doing an eigendecomposition of this matrix that eventually 2/3 of the population of America is living outside California.

3. In the highly competitive chocolate bar market there are two dominating Brands: Nestle & Cadbury. At any time *n* the sample space of possible outcomes is (*N, C*) where *N* = % of market share that eats Nestle bars in a particular year and *C*, = % that which eats Cadbury's products. It is found that the chances of someone switching their preferences from Cadbury bars to Nestle bars is 0.2 and the chances of someone switching the other way are 0.4.
    a. What is the Transition matrix for this system?
    b. Show that the Transition matrix has an eigenvalue of 1 and find the other eigenvalue.
    c. What is the state of the market for large *n*?

4. The Swiss chocolatier Lindt decides to enter the market described in Q2 following advice by consultants that they can take 50% of the market in 5 years by taking 20% of Cadbury's share of the market in any year *n* and 30% of Nestle's.
    a. Draw a Transition diagram for the above situation.

b. Investigate, assuming that we start from the state of the market in Q2c and that other switching percentages remain the same[1] whether (provided people switch to Lindt and don't switch back) this claim is true or hype.

5. In the market with two products A and B, shown in the Transition Diagram (Figure Q4). There is a natural 'churn' rate between the products given by *z* and a further bias *a* in favour of buying product A over B (because of taste, value, quality).

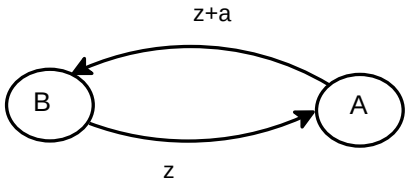

Figure Q4

a. Show that the Transition matrix for the market is given by

$$\begin{array}{ccc} & \text{From} & \\ & \text{A} & \text{B} \\ \text{To} \quad \begin{array}{c}\text{A}\\ \text{B}\end{array} & \begin{bmatrix} 1-z-a & z \\ a+z & 1-z \end{bmatrix} & \end{array}$$

b. Show that the steady state in the market is given by:

$$X_1 = \frac{1}{z+a}\begin{bmatrix} z \\ z+a \end{bmatrix}$$

[1] i.e. p(Cadbury to Nestle)=0.2, p(Nestle to Cadbury)=0.4