

# CA659 Mathematical Models/Computational Science

## In-Class Exercise 3

### Exercises on Leslie Matrices

1. In a population of an imaginary organism that lives for two years, the average number of births for one-year-olds is  $1/3$  and that for two-year-olds is 4. The survival probability from one to two years old is  $2/3$ . Death is certain after 3 years.
  - a. Set up a Leslie matrix for the population.
  - b. Find the long-term growth rate and the stable age distribution for this population.
2. An unlucky form of insect lives no more than two years, with the average number of births for one-year-olds of 1 and that for two-year-olds of 2. The survival probability from one to two years old is  $1/2$ . Death is certain after 3 years.
  - a. Set up a Leslie matrix for the population.
  - b. Determine (in stability terms) whether this insect will survive or die out.
  - c. If it survives, find the long-term growth rate and the stable age distribution for its population.
3. So far, we have dealt with  $2 \times 2$  matrices where the determinant was quite simple to determine. For  $3 \times 3$  Matrices, the method to find the determinant becomes a bit more laborious. In order to calculate  $\det(A)$  for a  $3 \times 3$  matrix we proceed as follows:

- a. For a matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

We have to start by taking a reference row or column. Taking

a reference row or column in this problem is a crucial step and with experience, you can make the problem easier and solve it in much less time.

1. Generally first row is taken as reference. If the matrix has a zero in its elements, then choose the reference row or column which has the most zeros,
2. There is a sign convention for the reference row or column according to which you have to do calculation. Note that this sign convention only for reference row or column. The sign convention is given below.

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

3. Select the first element from the reference row or column and cross out other elements from the row and column which the selected element is in. Calculate the product of the first element of the reference row/column and the determinant of  $2 \times 2$  matrix which is left after crossing out elements in above step. Calculate the  $2 \times 2$  determinant with

selected reference element and corresponding sign for that reference element. If you are taking first row as reference, then for the first element is given by:

$$+a_{11}X \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

4. Proceed like this for the other two elements of the first row.

b. Let us take an example of this:

For a matrix like so, let us take the first row to be the reference row, thus

$$\begin{bmatrix} 1 & 5 & 3 \\ 2 & 4 & 7 \\ 4 & 6 & 2 \end{bmatrix}$$

$\begin{bmatrix} 1 & \blacksquare & \blacksquare \\ \blacksquare & 4 & 7 \\ \blacksquare & 6 & 2 \end{bmatrix}$  The first element of the determinant is computed as follows:

$$= +1X \begin{vmatrix} 4 & 7 \\ 6 & 2 \end{vmatrix} = +1X [4X2 - 7X6] = -34$$

$\begin{bmatrix} \blacksquare & 5 & \blacksquare \\ 2 & \blacksquare & 7 \\ 4 & \blacksquare & 2 \end{bmatrix}$  The second element of the determinant is computed as follows:

$$= -(5)X \begin{vmatrix} 2 & 7 \\ 4 & 2 \end{vmatrix} = -5X [2X2 - 4X7] = +120$$

$\begin{bmatrix} \blacksquare & \blacksquare & 3 \\ 2 & 4 & \blacksquare \\ 4 & 6 & \blacksquare \end{bmatrix}$  The final element of the determinant is computed as follows:

$$= +3X \begin{vmatrix} 2 & 4 \\ 4 & 6 \end{vmatrix} = +3X [2X6 - 4X4] = -12$$

So the final determinant is  $-34+120-12=+74$

4. For a certain population, the Leslie matrix is given by:

$$L = \begin{bmatrix} 0 & 7 & 6 \\ 1/4 & 0 & 0 \\ 0 & 1/2 & 0 \end{bmatrix}$$

a. Show that this matrix has an eigenvalue of  $3/2$ . Using Gaussian Elimination determine the corresponding eigenvector

b. If are the dominant eigenpair, show that the population will be stable with distribution 18:3:1.

5. A slightly less unlucky form of insect than Q2 lives no more than three years, with the average number of births for two-year-olds and three-year-olds of 2.5. The survival probability from one to two years old is  $1/3$  and that of two to three year olds is  $1/4$ .

a. Set up a Leslie matrix for the population.

b. Determine (in stability terms) whether this insect will survive or die out.

c. If it survives, find the long-term growth rate and the stable age distribution for its population.