Solutions to some, though not all, of the questions are provided here.

Q1. [*Solution not provided*]

Q.2 [*Solution not provided*]

Q.3 *Solution:*

(**First part of question**) When, as in this part, n is large it is permissible to take σ = s. Hence we can apply the "formula" for the 99% confidence interval of the population mean μ: ($\bar{x}$ - 2.58σ /√n, $\bar{x}$ + 2.58σ / √n). So, required interval = (12.9 - 2.58(3.2)/11, 12.9 + 2.58(3.2)/11) = (12.15, 13.65)

(**Second part of question**) In this part, the sample size is small so that ($\bar{x}$ - μ)/(s / √n) ~ t_{n-1}, that is, T_8 = 3(12.9 - μ)/3.2 ~ t_8. So we need β such that P[-β < 3(12.9 - μ)/3.2 < β] = 0.99. Equivalently, we have

1 - P[3(12.9 - μ)/3.2 < -β or 3(12.9 - μ)/3.2 > β] = .99 or P[3(12.9 - μ)/3.2 < -β or 3(12.9 - μ)/3.2 > β] = 0.01. Hence, from Table 7 of notes, β = 3.355. Therefore, we have

P[-3.355 < 3(12.9 - μ)/3.2 < 3.355] = 0.99.

Hence, P[(-3.355)(3.2)/3 < 12.9 - μ < (3.355)(3.2)/3] = 0.99 =>

P[12.9 - (3.355)(3.2)/3 < μ < 12.9 + (3.355)(3.2)/3] = 0.99 =>

P[9.32 < μ < 16.48] = 0.99.

Q.4 *Solution:*

First calculate the sample mean and variance, $\bar{x}$ = $\Sigma x_i/10$ = 18.345 and $s^2 = \Sigma(x_i - \bar{x})^2/9$ = 8.087.

(**First part of question**) In this part, the sample size is small and σ is unknown so that ($\bar{x}$ - μ)/(s / √n) ~ t_{n-1}, that is, T_9 = (√10)(18.345 - μ)/√8.087 ~ t_9. So we need β such that P[-β < (√10)(18.345 - μ)/√8.087 < β] = 0.95. Equivalently, we have

1 - P[(√10)(18.345 - μ)/√8.087 < -β or (√10)(18.345 - μ)/√8.087 > β] = .95 or P[(√10)(18.345 - μ)/√8.087 < -β or (√10)(18.345 - μ)/√8.087 > β] = 0.05. Hence, from Table 7 of notes, β = 2.262. Therefore, we have

P[-2.262 < (√10)(18.345 - μ)/√8.087 < 2.262] = 0.95.

Hence, P[(-2.262)(√8.087)/√10 < 18.345 - μ < (2.262)(√8.087)/√10] = 0.95 =>

P[18.345 - (2.262)(√8.087)/√10 < μ < 18.345 - (2.262)(√8.087)/√10] = 0.95 =>

P[16.31 < μ < 20.38] = 0.95

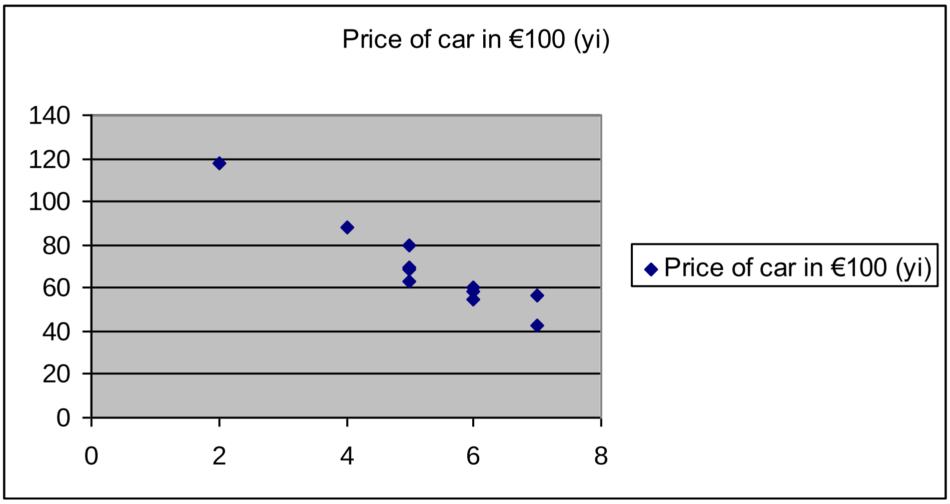
(**Second part of the question**) Here, σ is known so we can use the "formula" for the 95% confidence interval of the population mean μ: ($\bar{x}$ - 1.96σ /√n, $\bar{x}$ + 1.96σ / √n).

So, required interval = (18.345 - 1.96(3)/√10, 18.345 + 1.96(3)/√10) = (16.49, 20.20).

Q.5 [*Solution not provided*]

Q.6 *Solution:*

(a) A scatter plot [Obviously would need to be done by hand in an exam):



From this plot a linear relationship does seem plausible.

(b) To compute the regression line one must apply the formulae

$$b = \frac{n\sum_{i=1}^{n} x_i y_i - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2} \quad , \quad a = \bar{y} - b\bar{x} \quad , \quad r = \frac{n\sum_{i=1}^{n} x_i y_i - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{\sqrt{(n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2)(n\sum_{i=1}^{n} y_i^2 - (\sum_{i=1}^{n} y_i)^2)}}$$

As in the lecture notes it is easiest to lay the calculations in tabular form. The formula for r is needed for part (b) but it’s simplest to include its elements in the table also.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	Sums	Means
x_i	5	7	6	6	5	4	7	6	5	5	2	58	5.3
y_i	80	57	58	55	70	88	43	60	69	63	118	761	69.2
x_i^2	25	49	36	36	25	16	49	36	25	25	4	326	--
$x_i y_i$	400	399	348	330	350	352	301	360	345	315	236	3736	--
y_i^2	6400	3249	3364	3025	4900	7744	1849	3600	4761	3969	13924	56785	--

Hence

$$b = \frac{n\sum_{i=1}^{n} x_i y_i - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2} = \frac{11(3736) - 58(761)}{11(326) - (58)(58)} = -13.7$$

$$a = \bar{y} - b\bar{x} = 69.2 - (-13.7)(5.3) = 141.4$$

Hence, the estimated price

for a 3 year old car is 141.4 – 13.7(3) = 141.4 – 41.1 = 100.3

and

for a 5.5 year old car is 141.4 – 13.7(5.5) = 66.1.

[***Note: Check calculations!***]

(c) The correlation coefficient is

$$r = \frac{n\sum_{i=1}^{n} x_i y_i - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{\sqrt{(n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2)(n\sum_{i=1}^{n} y_i^2 - (\sum_{i=1}^{n} y_i)^2)}} = \frac{11(3736) - 58(761)}{\sqrt{(11*326 - 58*58)*(11*56785 - 761*761)}} = -0.957$$

Hence, $r^2 = 0.916$ which implies that 91.6% of the variation in price is accounted for by the age.