

Summary of relevant probability distributions with examples of table usage and indication of application areas

Distribution	Example(s)	Applications
Normal distribution If X is $N(\mu, \sigma^2)$ or $N(\mu, \sigma)$	Suppose that 10 per cent of the probability for a certain distribution which is $\sim N(\mu, \sigma^2)$ is below 60 and that 5 per cent is above 90. What are the values of μ and σ ? We have $\text{Prob}(X \leq 60) = 0.1, \text{Prob}(X \leq 90) = 0.95$ By “Key Fact” this implies $\text{Prob}((X - \mu)/\sigma \leq (60 - \mu)/\sigma) = 0.1$ or $\text{Prob}((X - \mu)/\sigma > (60 - \mu)/\sigma) = 0.90$ and $\text{Prob}((X - \mu)/\sigma \leq (90 - \mu)/\sigma) = 0.95$ or $\text{Prob}((X - \mu)/\sigma > (90 - \mu)/\sigma) = 0.05$ From Table 4 $(60 - \mu)/\sigma = -1.28$ and $(90 - \mu)/\sigma = 1.65$ Solving we get $\mu = 73.1$ and $\sigma = 10.2$, approximately	Key fact: $\frac{X - \mu}{\sigma} \sim N(0, 1)$ The main result, when sampling, is $\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$ When dealing with proportions we have, similarly and using the Central Limit Theorem $\frac{p - \mu_p}{\sigma_p} \rightarrow N(0, 1) \text{ as } n \rightarrow \infty$
Student’s T (t) distribution t_n	(a) Let T have a t distribution with 10 <i>degrees of freedom</i> . Find $\text{Prob}(T > 2.228)$ from Table 7 <u>Answer:</u> $\text{Prob}(T > 2.228) = \text{Prob}(-2.228 < T \text{ or } T > 2.228)$ so we	Small samples from Normal Distribution $n < 30$, σ unknown

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	<p>can use the “two-sided test” element of table with $v = 10$. We find $\text{Prob}(T > 2.228) = 0.05$.</p> <p>(b) If T has a t distribution with 20 degrees of freedom then, $\text{Prob}(T > 2.086) = 0.05$.</p> <p>(c) If T has a t distribution with 120 degrees of freedom then, $\text{Prob}(T > 1.98) = 0.05$.</p> <p>(d) If finally T has a t distribution with infinite degrees of freedom we get the Normal distribution so that $\text{Prob}(T > 1.96) = 0.05$.</p>	$\frac{\bar{x} - \mu}{s/\sqrt{n}} \sim t_{n-1}$
<p>Chi-Squared distribution. χ_v^2, $v =$ degrees of freedom</p>	<p>Let X be χ_{10}^2. Then by Table 8¹ $\text{Prob}(3.25 \leq X \leq 20.5) =$ $\text{Prob}(X \geq 3.25) - \text{Prob}(X \geq 20.5)$ $= 0.975 - 0.025 = 0.95$</p>	<p>Later, we will use the fact that $\frac{(n-1)s^2}{\sigma^2} \sim \chi_{n-1}^2$ to obtain <i>Confidence Intervals</i> for σ^2 and we will have some other uses for χ_v^2 also.</p>
<p>F distribution $F_{v_1, v_2} = \frac{\chi_{v_1}^2 / v_1}{\chi_{v_2}^2 / v_2}$</p>	<p>Relevant table is Table 9. We omit an example of using this table for the moment.</p>	<p>Useful when dealing with situation of taking samples (sizes n_1 and n_2) from 2 different populations (standard deviations σ_1 and σ_2). It turns out that $\frac{s_1^2 / \sigma_1^2}{s_2^2 / \sigma_2^2} \sim F_{n_1-1, n_2-1}$</p>

¹ Note: Mistake in Table 8 on Web pages– have mistakenly reproduced Page 17 twice. On the correct page 18, for $v = 10$, then $\text{Prob}(\chi^2 > 20.4832) = 0.025$. There are some copies of the tables in the library that you can check.