CA200 – Quantitative Analysis for Business Decisions

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3.2.1 Criterion of maximising "Expected Value"

Example 1: A Concession problem

The following **Payoff Matrix** is given:

	States/events	
Action	Cold weather (p = 0.3)	Warm weather (p = 0.7)
a_1 : sell cola	€1,500	€5,000
a_2 : sell coffee	€4,000	€1,000

The business owner must decide whether to sell cola or coffee. The weather is not under the business owner's control!

Based on the payoff matrix we can calculate the "expected value" of each action and use its maximum as the decision criterion:

	States/events		
Action	Cold weather (p = 0.3)	Warm weather (p = 0.7)	**Choice Criterion** Expected Values of actions
a_1 : sell cola	€1,500	€5,000	$E(a_1)$ =1500×(0.3) + 5000×(0.7) = €3,950
a_2 : sell coffee	€4,000	€1,000	$E(a_2)$ =4000×(0.3) + 1000×(0.7) = €1,900

Therefore, in this case and using this criterion the decision is to "SELL COLA".

Example 2: Two projects are being considered for which project data have been estimated, enabling expected values (EV) to be calculated as follows.

	Project A					Project B				
	€		p		EV	€		p		EV
Optimistic outcome	6000	×	0.2	=	1200	6500	×	0.1	=	650
Most likely outcome	3500	×	0.5	=	1750	4000	×	0.6	=	2400
Pessimistic outcome	2500	×	0.3	=	750	1000	×	0.3	=	300
Project EV					3700					3350

On the basis of EV, Project A would be preferred.

Notes:

(1) Although Project A's EV is €3700, this value would only be achieved in the long run over many similar decisions – extremely unlikely circumstances.

(2) If project A was implemented, any of the three outcomes could occur, with the indicated values.

Summary of "Expected Value" advantages and disadvantages:

Advantages	Disadvantages
Simple to understand and calculate	-
Represents whole distribution by a single figure	Representation of whole distribution by a single figure means that other characteristics are ignored (e.g. range)
Takes account of the expected variability of all outcomes	Makes the assumption is risk neutral. (see * following this table)

* The assumption of being "risk neutral" would mean that for the data

	Project X					Project Y				
	€		p		EV	€		p		EV
Optimistic outcome	18000	×	0.25	=	4500	6000	×	0.2	=	1200
Most likely outcome	20000	×	0.5	=	10000	18000	×	0.6	=	10800
Pessimistic outcome	22000	×	0.25	=	5500	40000	×	0.2	=	8000
Project EV					20000					20000

the projects would be ranked the same. However, it seems likely that different people would decide differently on the project to choose depending on their attitude to risk.

Example 3: (***Cf §1.4.2 - Use of Probability, Expected Value in making a decision***)
A distributor buys perishable goods for €2 per item and sells them at €5. Demand per day is uncertain and items unsold at the end of the day represent a write-off because of perishability. If the distributor understocks then he/she loses profit that could have been made. A 300-day record of past activity is as follows:

Daily demand (units)	No. of days	P (probability)
10	30	0.1 (= 30/300)
11	60	0.2 (= 60/300)
12	120	0.4 (= 120/300)
13	90	0.3 (= 90/300)
Σ (column sums)	300	1.0

What level of stock should be held from day to day to maximise profit? [*The answer turns out to be 'to stock 12 units per day'*]

Solution: We proceed by calculating the Conditional Profit (CP) and the Expected Profit (EP).

CP = profit that would be made at any particular combination of stock and demand; for example, if 13 articles were bought and demand was 10 then

CP = Total sale price – Total purchase cost = 10×5 – 13×2 = €24

EP = CP × probability of demand so that in the above example

EP = €24×0.1 = €2.4

We must do these calculations for all combinations of stock and demand, as follows:

		Stock Options							
		10		**11**		**12**		**13**	
Demand	p	CP €	EP €	CP €	EP €	CP €	EP €	CP €	EP €
10	0.1	30	3	28	2.8	26	2.6	24	2.4
11	0.2	30	6	33	6.6	31	6.2	29	5.8
12	0.4	30	12	33	13.2	36	14.4	34	13.6
13	0.3	30	9	33	9.9	36	10.8	39	11.7
	1.0		30		32.5		**34**		33.5

Remember, the distributor can only decide on what level of stock to hold each day. It is clear from the above table that the stock level that gives the best average profit is 12.

3.2.2 Equivalence of criteria of maximising EV & minimising EOL

We introduce the abbreviation

EOL = Expected opportunity loss

Example 1: An investment problem

We are given the following data:

	States (financial conditions)	
Action	Condition 1 p = 0.4	Condition 2 p = 0.6
A	€50,000	– €10,000
B	€15,000	€60,000
C	€100,000	€10,000

(a) **First** if we were to apply the criterion of "maximise expected profit" we would calculate

(EV(A) =) $E(A) = 50000 \times 0.4 - 10000 \times 0.6 = 14000$

$E(B) = 15000 \times 0.4 + 60000 \times 0.6 = 42000$

$E(C) = 100000 \times 0.4 + 10000 \times 0.6 = 46000$

So we would decide on action C.

(b) Next we apply the criterion of "minimise EOL".

To do this the procedure is to first calculate the "Regret Matrix" by subtracting each payoff from the best in its column:

	States (financial conditions)	
Action	Condition 1 p = 0.4	Condition 2 p = 0.6
A	€50,000	€70,000
B	€85,000	€0
C	€0	€50,000

Interpretation: For example, if condition 2 occurred and you had chosen action C then you would have lost the opportunity of gaining €50,000 more by selecting the best action for that state (namely Action B).

Of course the problem is that we don't know in advance which condition will occur.

Next calculate EOL for each action from the Regret Matrix as

$EOL(A) = 0.4(50{,}000) + 0.6(70{,}000) = €62{,}000$
$EOL(B) = 0.4(85{,}000) + 0.6(0) = €34{,}000$
$EOL(C) = 0.4(0) + 0.6(50{,}000) = €30{,}000$

Finally, choose the action that **minimises** EOL. It is seen that this is the same action as for maximising the expected profit.

3.2.3 Value of Perfect Information

It is often worthwhile to ask whether we should get more information before making a decision, and how much is such additional information worth.

The extreme case is to have "Perfect Information" and to determine what this perfect information is worth.

We can calculate the "Expected Value of Perfect Information *(EVPI)*" as follows, using the investment problem of the previous section to illustrate.

We had previously found (when uncertain as to which condition would hold) that E(A) = €14,000, E(B) = €42,000 and E(C) = €46,000.

On the other hand, if the condition were known in advance (certainty) the choice would be:

Condition 1 → choose *C* (100,000)

Condition 2 → choose *B* (60,000)

We have, to summarise,

Expected value under <u>certainty</u>	**Expected value under <u>uncertainty</u>**
0.4×100,000 + 0.6×60,000 = €76,000	E(C) = €46,000 (*as before*)

Then, we define

EVPI = Expected value under certainty - Expected value under uncertainty

=>

EVPI = 76,000 – 46,000 = €30,000

Note: EVPI = Min *EOL* (*EOL(C)* = €30,000)

<u>Example 1</u>: Consider again Example 3 of section 3.2.1 "A distributor buys perishable goods for €2 per item and sells them at €5. Demand per day is uncertain and it …" Assume now that it is possible for the distributor to buy market research information that was perfect, that is it would enable him to forecast the exact demand on any day so that he could stock up accordingly. How much would the distributor be prepared to

pay for such information? The procedure is to compare the profit with perfect information with the optimum EP (€34), previously calculated.

<u>Solution</u>: When we have perfect information we have,

When demand is 10, stock 10 => Profit = (10×3)×0.1 = €3.0

When demand is 11, stock 11 => Profit = (11×3)×0.2 = €6.6

When demand is 12, stock 12 => Profit = (12×3)×0.4 = €14.4

When demand is 13, stock 13 => Profit = (13×3)×0.3 = €11.7

TOTAL (expected value under certainty) = €35.7

We conclude that EVPI = €35.7 - €34 = €1.7 that is, the distributor could pay up to this amount for the information.

Note: In reality, information will **never be perfect** but it can sometimes be worth getting some additional imperfect information. The next example illustrates this.

<u>Example 2</u>: A company is considering launching a new product. Various **prior** estimates have been made as follows from which we have calculated the expected value in the usual way:

Market state	**p**	**Profit or Loss**	**Expected Value (EV) €**
Good	0.2	60000	12000
Average	0.6	40000	24000
Bad	0.2	-40000	-8000
			EV = 28000

Note: In the absence of any other information EV = 28,000 and using this criterion the company would launch.

However, in order to have more information on which to base its decision the company is considering whether to commission a market research survey at a cost of €1000. The agency concerned produces reasonably accurate, but not perfect information as follows,

Likely actual market state	**Market research Agency survey findings**		
	Good	**Average**	**Bad**
Good	60%	30%	-
Average	40%	50%	10%
Bad	-	20%	90%

For example, if the agency reports the market to be average there is a probability of 0.2 that it is, in fact, bad.

Solution: The first step is to calculate the **posterior** probabilities (using our conditional probability rule $p(X \mid Y) = \frac{p(X \text{ and } Y)}{p(Y)}$) after the market research results are available. For example,

p(Market is good and Survey predicts good) = 0.2×0.6 = 0.12 [*call this GG*]

In total, we have

Prior probability	**Market state**	**Market Research Results**		**Posterior probability**
0.2	Good	Good	0.6	GG = 0.12
		Average	0.4	GA = 0.08
0.6	Average	Good	0.3	AG = 0.18
		Average	0.5	AA = 0.30
		Bad	0.2	AB = 0.12
0.2	Bad	Average	0.1	BA = 0.02
		Bad	0.9	BB = 0.18
Total probability				1.0

From this table we can summarise,

p(**Survey** will show **G**ood) = G**G** + A**G** = 0.30

p(**Survey** will show **A**verage) = G**A** + A**A** + B**A** = 0.40

p(**Survey** will show **B**ad) = A**B** + B**B** = 0.30

Assuming that the product will be launched if the survey predicts a good or average market the following table can be prepared:

Survey results	**Decision**	**Actual market**	**Posterior probabilities**	**Profit or Loss**	**EV of Profits/Losses**
Good	Launch	Good	0.12	60000	7200
		Average	0.18	40000	7200
Average	Launch	Good	0.08	60000	4800
		Average	0.30	40000	12000
		Bad	0.02	-40000	-800
Bad	No Launch	Bad	0.30	0	0
			1.0		€30400

Finally, we can calculate the **value** of the *imperfect* information as €(30,400 – 28,000) = €2400. So, as the survey cost is €1000 it does appear worthwhile to do the research although the gain is not that great.

3.2.4 Other decision criteria – Utility idea

Other criteria may be preferred by

- risk averters
- risk takers

Example 1:- Fire insurance

Action	States/Events		Expected Value
	Fire, p = 0.0002	**No fire p = 0.9998**	
Insurance	-€300	-€300	-€300
No insurance	-€50000	0	-€10

While 'No insurance' has a higher expected value, most people take insurance.

It appears that most people attach a **utility** (other than just monetary gain) to this issue. For example, might specify

Insurance premium €300:- utility value = – 1
Loss of property €50,000:- utility value = –100,000

Then can set up a "Utility Matrix":

Action	States/Events		Expected Value
	Fire, p = 0.0002	**No fire p = 0.9998**	
Insurance	-1	-1	-1
No insurance	-100000	0	-20

Then, the expected utility value is more favourable for the decision to taking out insurance.

3.3 Decision making under Uncertainty (Probs of states/events not known)

We consider an investment example and introduce a number of **different** decision criteria that might be used.

Example 1:

	Economic conditions (states)		
Investment	**High Growth**	**Moderate Growth**	**Low Growth**
Shares	€10000	€6500	- €4000
Bonds	€8000	€6000	€1000
Savings	€5000	€5000	€5000

Criterion 1 (Laplace) – assume all states are equally likely

E(Shares) = 10,000(1/3) + 6,500(1/3) + (–4,000)(1/3) = €4,167

E(Bonds) = €5,000

E(Savings) = €5,000

Therefore choose **either** Bonds **or** Savings

Criterion 2 (**Maximin – Best of the worst**) – decision maker is **conservative**.

Compare minimum returns for each alternative.

Shares - -€4,000

Bonds - €1,000

Savings - €5,000

Criterion indicates to choose maximum (Savings)

Criterion 3 (**Maximax - Best of the best**) – decision maker is **optimistic**.

Compare maximum returns for each alternative.

Shares - €10,000

Bonds - €8,000

Savings - €5,000

Criterion indicates to choose maximum (Shares)

Criterion 4 (Hurwicz – **compromise** between maximin and maximax)

It involves a degree of optimism. The **coefficient of optimism = α** where

0 (pessimistic) ≤ α ≤ 1 (optimistic)

We calculate (maximum payoff) * (α) + (minimum payoff) * (1 – α)

For example, for α = 0.6

Shares: 10,000 x 0.6 + (- 4,000) x 0.4 = €4,400

Bonds: 8,000 x 0.6 + 1,000 x 0.4 = €5,200

Savings: 5,000 x 0.6 + 5,000 x 0.4 = €5,000

So, in this case, the decision would be to invest in Bonds.

Note: We can see that α = 0 → maximin, α = 1 → maximax

Criterion 5 (Minimax regret)

We form the "Regret Matrix" (see before about EOL). For the example, we have

	Economic conditions (states)		
Investment	**High Growth**	**Moderate Growth**	**Low Growth**
Shares	€0	€0	€9000
Bonds	€2000	€500	€4000
Savings	€5000	€1500	€0

The maximum "regrets" are €9000, €4000 and €5000. We choose the minimum of these i.e. choose Bonds.

3.4 Sequential decisions – Decision Trees (See …3B …)