CA200 – Quantitative Analysis for Business Decisions

File name: CA200_Section_02B_Probability

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2.5 Permutations ("order matters")

Permutations have to do with arranging a set of items in a particular order, and especially with working out the total number of possible arrangements. In general, this has to do with the very important topic of symmetry in Maths but for our purposes we just need a few straightforward results.

Example 1: (i) Restaurant A offers a choice of 2 starters, 3 main courses and 3 desserts. How many different meals are available?

Answer: We use a table to "spell out" the 18 = 2x3x3 different meals ("a" to "r"):

Meal	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r
St	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2
Ma	1	1	1	2	2	2	3	3	3	1	1	1	2	2	2	3	3	3
De	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3

(ii) Restaurant B offers a choice of 4 starters, 5 main courses and 4 desserts. How many different meals are available?

Answer: It would be tedious to list all the combinations explicitly but we see that the total is 4x5x4 = 80 different meals.

Example 2: (i) A transport manager has to plan routes so as to make deliveries to 3 different customers X, Y and Z. What are the different possible routes and how many are there in total. Assume that the customers can be delivered to in any order.

Answer: We can again use a table:

Route	**1**	**2**	**3**	**4**	**5**	**6**
First delivered to	X	X	Y	Y	Z	Z
Second delivered to	Y	Z	X	Z	X	Y
Third delivered to	Z	Y	Z	X	Y	X

The total number of different routes is 6 = 3! (factorial 3).

(ii) What if there are 10 different customers?

Answer: There are too many routes to list explicitly but the total number of different routes is 10! = 10x9x8x7x6x5x4x3x2x1 = 3628800.

Note of interest: There is a very well known problem in Operations Research called the Travelling Salesman problem which is stated as follows:

> *A salesman has a certain number of cities that he must visit. He knows the distance (or time or cost) between each pair of cities. His problem is to select a route that starts at his home city, passes through each city once and only once, and returns to his home city in the shortest possible distance (or time or cost).*

We can see that the transport manager might be concerned with this type of problem. If there are just three cities to visit then there are 3! = 6 routes so that it is possible to simply calculate the distance (or time or cost) of each and pick the "cheapest" one. However, it is clear that this approach is impossibly inefficient for a large number of cities. We may come back to this problem later on in the course.

Example 3: ***Dealing with groups of items within a list***

(i) A company has 4 training officers A, B, C and D and it is required to assign one to each of 2 training sections X and Y. In how many different ways may the 4 officers be assigned to the 2 sections?

(ii) What if there were 7 officers and 3 sections?

(iii) What if there were n officers and r sections?

Solution:

(i) It is convenient to use a table to set out the different possible assignments:

Assignment	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Section X	A	A	A	B	B	B	C	C	C	D	D	D
Section Y	B	C	D	A	C	D	A	B	D	A	B	C

We see there are 4 ways of allocating to section X and for each of these there are then 3 ways of allocating to section Y. So, the answer is 4*3 = 12.

(ii) This is a bit long to list all the different possibilities so instead we observe there are 7 ways of assigning to the first section and for each of these there are 6 ways of assigning to the second section, which leaves 5 ways of assigning to the 3^{rd} section. So the answer is 7*6*5 = 210.

Note: We can also write this as $\frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7*6*5*4*3*2*1}{4*3*2*1} = 7*6*5 = 210$

This form is useful when coming to answer part (iii).

(iii) The answer is simply $\frac{n!}{(n-r)!}$

This is the number of permutations and is sometimes denoted by ${}^{n}P_{r}$.

2.6 Combinations ("order does not matter")

Example 1: 6 apprentices A, B, C, D, E and F have to be paired into twos for an exercise. In how many ways can this be done?

Note: A key point is that the order does not matter. For example, the pair {A, B} is the same as the pair {B, A}.

Solution: We can list the pairs explicitly for this small problem,

AB AC A D AE AF [*A with each of the others*]

BC BD BE BF [*B with each of the others except A as AB is already included*]

CD CE CF [*C with others apart from AC and BC, already included*]

DE DF [*D with others apart from AD, BD and CD, already included*]

EF [*Only pair not already counted*]

We can see there is a total of 5 + 4 + 3 + 2 + 1 = 15 pairings.

Note: There is a general formula for calculating the number of ways of selecting k objects from n objects. This formula is

$$^{n}C_{k} \quad \binom{n}{k} = \frac{n(n-1)\dots(n-k+1)}{k(k-1)\dots1} = \frac{n!}{k!(n-k)!}$$

=

This term is called the "binomial coefficient".

<u>Example 2</u>: A class of 60 students are to be formed into groups of five for a continuous assessment project. In how many ways can this be done?

<u>Solution</u>: We apply the formula above with n = 60 and k = 5. This gives

$$\frac{60!}{5!(60-5)!} = \frac{60!}{(5!)(55!)} = \frac{60*59*58*57*56}{5*4*3*2*1} = 59*58*57*28 = 5461512$$

2.7 Expected value

This will be covered in Section 3 "Decision Theory".

2.8 Value of Perfect Information

This will be covered in Section 3 "Decision Theory".

2.9 Alternative decision rules

This will be covered in Section 3 "Decision Theory".

2.10 Summary

We introduced the notion of probability as relative frequency though often in business decision-making one has to rely on subjective probabilities based on judgement.

We introduced the basic rules of probability and the notions of mutually exclusive events and independent events.

Bayes' Rule [*p(A|B)=p(A)p(B|A)/p(B)*] allows one to find conditional probability p(A|B) when given its inverse p(B|A).

A permutation is an ordered arrangement whereas a combination is an arrangement without regard to order.

2.11 Exercises

(**E2.1**) A card is drawn from a shuffled pack of 52 cards. What is the probability of drawing a ten or a spade?

(**E2.2**) Records of service requests at a garage and their probabilities are as follows:

Daily Demand	**Probability**
5	0.3
6	0.7

Daily demand is independent (e.g. to-morrow's demand is independent of to-day's demand).

What is the probability that over a two-day period the number of requests will be

a) 10 requests, b) 11 requests and c) 12 requests?

(**E2.3**) Analysis of a questionnaire completed by holiday makers showed that 0.75 classified their holiday as good at resort Costa Lotta. The probability of hot weather in this resort is 0.6. If the probability of regarding the holiday as good given hot weather is 0.9, what is the probability that there was hot weather if a holiday maker considers his holiday good?