CA200

Example 1: A company involved in the assembly and distribution of printers in concerned with two types – laser and inkjet. Assembly of each laser printer takes two hours, while each inkjet printer takes one hour to assemble, and the staff can provide a total of 40 person-hours of assembly time per day. In addition, warehouse space must be available for the assembly and distribution of the printers, 1 square metre for each laser printer and 3 square metres for each inkjet printer; the company has a total of 45 square metres of storage space available for assembled printers each day. Laser printers can be sold for a profit of €30 per unit and inkjet printers earn a profit of €25 each, but the market in which the company is operating can absorb a maximum of 12 laser printers per day. (There is no such limitation on the market for inkjet printers). Formulate this as a linear programming problem and determine, using the simplex method, the number of each type of printer the company should assemble and distribute in order to maximise daily profit.

Solution: Let X_1 = number of laser printers assembled
X_2 = number of inkjet printers assembled

Maximise $30X_1 + 25X_2$
Subject to $2X_1 + X_2 <= 40$
$X_1 + 3X_2 <= 45$
$X_1 <= 12$
$X_1 , X_2 >=0$

NOW
- Can use R or AMPL or other to solve
- In this case (as 2 decision variables only) can also solve graphically.

Answer: Optimal. Solution $X_1^* = 12$(# of laser printers), $X_2^* = 11$ (# of inkjet printers).
Total daily profit = €635

Example 2:

A machine tool company conducts a job-training program for machinists. Trained machinists are used as teachers in the program at a ratio of one for every ten trainees. The training program lasts for one month. From past experience it has been found that out of ten trainees hired, only seven complete the program successfully (the unsuccessful trainees are released).

Trained machinists are also needed for machining and the company's requirements for the next three months are as follows:

January	100
February	150
March	200

In addition, the company requires 250 trained machinists by April. There are 130 trained machinists available at the beginning of the year.

Payroll costs per month are:

Each trainee	€400
Each trained machinist (machining or teaching)	€700
Each trained machinist idle (Union forbids firing them!)	€500

Set up the linear programming problem (LPP) that will produce the minimum cost hiring and training schedule and meet the company's requirements. Then solve the LPP using an appropriate software R, AMPL or 'solver'. Comment on what you find and try changing some of the values to see what happens to the solution.

Solution:

Each trained machinist can:

(i) work a machine
(ii) teach
(iii) stay idle

The number of trained machinists is fixed; therefore the decision variables are the number teaching each month and the number idle each month.

Let X_1 = number of trained machinists teaching in January
X_2 = number of trained machinists idle in January
X_3 = number of trained machinists teaching in February
X_4 = number of trained machinists idle in February
X_5 = number of trained machinists teaching in March

X_6 = number of trained machinists idle in March

Each month the total number of trained machinists = number machining + number teaching + number idle.

January: $100 + X_1 + X_2 = 130 \rightarrow X_1 + X_2 = 30$

February: $150 + X_3 + X_4 = 130 + 7X_1 \rightarrow 7X_1 - X_3 - X_4 = 20$
(Note that each machinists teaching trains 10 with 70% success)

March: $200 + X_5 + X_6 = 130 + 7X_1 + 7X_3 \rightarrow 7X_1 + 7X_3 - X_5 - X_6 = 70$

April: 250 trained machinists required in April:
$130 + 7X_1 + 7X_3 + 7X_5 = 250 \rightarrow 7X_1 + 7X_3 + 7X_5 = 120$

The cost of machinists operating is constant and therefore not required in the objective function.

Relevant costs are:

(i) cost of training program i.e. cost of trainees + cost of teachers
(ii) cost of idle machinists

Objective function (cost minimisation):

Minimise $Z = 400\,(10X_1 + 10X_3 + 10X_5)$ (trainees)
$+ 700\,(X_1 + X_3 + X_5)$ (teachers)
$+ 500\,(X_2 + X_4 + X_6)$ (idle machinists)

Complete problem:

Minimise $Z = 4700X_1 + 500X_2 + 4700X_3 + 500X_4 + 4700X_5 + 500X_6$

Subject to
$X_1 + X_2 = 30$
$7X_1 - X_3 - X_4 = 20$
$7X_1 + 7X_3 - X_5 - X_6 = 70$
$7X_1 + 7X_3 + 7X_5 = 120$

$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0$